Indian Statistical Institute, Bangalore
B. Math.
First Year, Second Semester
Analysis II

Back paper Examination — Date : June 11, 2018
Maximum marks: 100 — Time: 3 hours

1. Let f be a bounded real valued function f on an interval $[a, b]$. Show that for any two partitions, the lower sum is always less than or equal to the upper sum. [15]

2. Show that every continuous function on a bounded interval $[a, b]$ is Riemann integrable. [15]

3. Fix $n \in \mathbb{N}$. Let $Y = \{(a_1, a_2, \ldots, a_n) : a_j \in \{0, 1\}\}$. Define $d : Y \times Y \to \mathbb{R}$ by $d((a_1, a_2, \ldots, a_n), (b_1, b_2, \ldots, b_n))$ equals the number of $\{j : a_j \neq b_j\}$. Show that d is a metric on Y. [15]

4. Let (X, d) be a compact metric space. Show that every sequence in X has a convergent subsequence. [15]

5. Let $h : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by

$$h(x, y) = \begin{cases} 0 & \text{if} \quad x \leq 0 \text{ or } x \geq y^2 \\ \frac{x}{y^2}(1 - \frac{x}{y^2}) & \text{if} \quad 0 < x < y^2, \end{cases}$$

Determine as to whether h is continuous at the origin or not. Compute partial derivatives $D_1 h(0, 0)$ and $D_2 h(0, 0)$ if they exist. [15]

6. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a function having continuous first order partial derivatives and satisfying $f(kx) = k^2 f(x)$ for all $k, x \in \mathbb{R}$. Show that for any $a \in \mathbb{R}^n$,

$$\sum_{i=1}^{n} a_i D_i f(a) = 2f(a).$$

[15]

7. Use the method of Lagrange multipliers to find the minimum of $2y_1^2 + y_2^2 + 2y_2^2$, subject to the condition $2y_1 + 3y_2 = 2y_3 + 13$. [15]